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On a characteristic property of the tent map

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As a generalization of a characteristic property of fundamental chaotic maps such as the tent map, a primitive chaos is defined by $(X, \{X_\lambda, \lambda \in \Lambda\}, \{f_{X_\lambda}, \lambda \in \Lambda\}))$ which satisfies the following property, where X is a set, $\{X_\lambda, \lambda \in \Lambda\}$ is a family of subsets of X which contains no empty set, and $\{f_{X_\lambda}, \lambda \in \Lambda\}$ is a family of maps $f_{X_\lambda} : X_\lambda \to X$ [1].

(P) For any infinite sequence $\omega_0, \omega_1, \omega_2, \ldots$ of sets, there exists an initial point $x_0 \in \omega_0$ such that $f_{\omega_0}(x_0) \in \omega_1$, $f_{\omega_1}(f_{\omega_0}(x_0)) \in \omega_2, \ldots$, where $\omega_i \in \{X_\lambda, \lambda \in \Lambda\}$ for each i.

The primitive chaos is closely related to general problems about determinism, causality, free will, and irreversibility [1], which are vital problems in science (e.g., refs. [2-26]).

Then, the following propositions present sufficient conditions for guaranteeing the primitive chaos.

Proposition 1 [1]. *If X is a nondegenerate Peano continuum, for any $\varepsilon > 0$, there exist finitely many nondegenerate Peano subcontinua $X_1, \ldots, X_n$ covering X such that dia $X_i < \varepsilon$, $i = 1, \ldots, n$. Then, for each i, for any positive integer n^i, for any n^i points $x_1^i, \ldots, x_{n^i}^i \in X_i$ and $y_1^i, \ldots, y_{n^i}^i \in X$, there exists a continuous surjection $f_{X_i} : X_i \to X$ such that $f_{X_i}(x_1^i) = y_1^i, \ldots, f_{X_i}(x_{n^i}^i) = y_{n^i}^i$, and they satisfy the property* (P).

Proposition 2 [27]. *If X is a Cantor set, for any positive integer n, there exist a partition $\{X_1, \ldots, X_n\}$ of X and maps $f_{X_i} : X_i \to X$, $i = 1, \ldots, n$, and they satisfy the property (P).*

Here, a nondegenerate space means the space that consists of more than one point, a Peano continuum is a locally connected continuum, and a continuum is a nonempty connected compact metric space. A Cantor set is any space that is homeomorphic to the Cantor middle-third set, and a space is a Cantor set if and only if it is a zero-dimensional, perfect compact metric space [28, Theorem 8.1].

Proposition 1 explains the reason why we are surrounded by diverse chaotic behaviors [29], and Proposition 2 implies the possibility of the Cantor set for a new recognition of natural phenomena.

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References

[1] Y. Ogasawara: J. Phys. Soc. Jpn. **79** (2010) 15002.

[2] P. S. Laplace: *Essai Philosophique sur les Probabilités* (Mme. Ve. Courcier, Paris, 1814) [in French].

[3] J. W. Gibbs: *Elementary Principles in Statistical Mechanics* (Yale Univ. Press, New Haven, 1902).

[4] S. A. Eddington: *The nature of the physical world* (J. M. Dent & Sons Ltd., London, 1928).

[5] A. Einstein, B. Podolsky, and N. Rosen: Phys. Rev. **47** (1935) 777.

[6] N. Bohr: Phys. Rev. **48** (1935) 696.

[7] D. Bohm: Phys. Rev. **85** (1952) 166; D. Bohm: Phys. Rev. **85** (1952) 180; D. Bohm: *Wholeness and the Implicate Order* (Routledge & Kegan Paul, London, 1980).

[8] H. Y. Carr and E. M. Purcell: Phys. Rev **94** (1954) 630.

[9] C. G. Jung and W. Pauli: *The Interpretation of Nature and the Psyche* (Bollingen Foundation Inc., New York, 1955).

[10] H. Everett, III: Rev. Mod. Phys. **29** (1957) 454.

[11] J. S. Bell: Physics **1** (1964) 195.

[12] S. Kochen and E. P. Specker: J. Math. Mech. **17** (1967) 59.

[13] J. Monod: *Le Hasard et la Nécessité: Essai sur la Philosophie Naturelle de la Biologie Moderne* (Éditions du Seuil, Paris, 1970) [in French].

[14] W-K. Rhim, A. Pines, and J. S. Waugh: Phys. Rev. B **3** (1971) 684.

[15] M. Jammer: *The Philosophy of Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1974).

[16] J. R. Searle: The Behavioral and Brain Sciences **3** (1980) 417.

[17] A. Aspect, P. Grangier, and G. Roger: Phys. Rev. Lett. **47** (1981) 460; A. Aspect, P. Grangier, and G. Roger: Phys. Rev. Lett. **49** (1982) 91; A. Aspect, J. Dalibard, and G. Roger: Phys. Rev. Lett. **49** (1982) 1804.

[18] I. Prigogine: *From Being to Becoming: Time and Complexity in the Physical Sciences* (W. H. Freeman and Company, San Francisco, 1981); I. Prigogine and I. Stengers: *Order Out of Chaos* (Bantam books, New York, 1984); I. Prigogine

(in collaboration with I. Stengers): *The End of Certainty: Time, Chaos, and the New Laws of Nature* (Free Press, New York, 1997).

[19] J. Ford: Phys. Today **36** (1983) 40.

[20] N. D. Mermin: Phys. Today **38** (1985) 38; N. D. Mermin: Rev. Mod. Phys. **65** (1993) 803.

[21] V. Ž. Vulović and R. E. Prange: Phys. Rev. A **33** (1986) 576.

[22] R. Penrose: *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics* (Oxford University Press, Oxford, 1989); R. Penrose: *Shadows of the Mind: a Search for the Missing Science of Consciousness* (Oxford University Press, Oxford, 1994).

[23] P. Coveney and R. Highfield: *The Arrow of Time: A Voyage Through Science to Solve Time's Greatest Mysteries* (W.H. Allen, London, 1990).

[24] A. Zeilinger: Nature **438** (2005) 743.

[25] J. Conway and S. Kochen: Found. Phys. **36** (2006) 1441; J. H. Conway and S. Kochen: Not. Am. Math. Soc. **56** (2009) 226.

[26] R. Colbeck and R. Renner: Nat. Phys. **8** (2012) 450.

[27] Y. Ogasawara and S. Oishi: arXiv:1203.0087v1.

[28] A. Illanes: *Hyperspaces* (Marcel Dekker Inc., New York, 1999).

[29] Y. Ogasawara and S. Oishi: J. Phys. Soc. Jpn. **81** (2012) 103001.

[30] K. Lewin (translated by F. Heider and G. M. Heider): *Principles of Topological Psychology* (McGraw-Hill Book Company, New York, 1936).

[31] J. Piaget: *Le Structuralisme* (Presses Universitaires de France, Paris, 1968) [in French].

[32] R. Thom: *Stabilité Structurelle et Morphogénèse* (InterEditions, Paris, 1977) [in French].

[33] S. B. Nadler, Jr.: *Continuum Theory* (Marcel Dekker Inc., New York, 1992).

[34] Y. Ogasawara and S. Oishi: J. Phys. Soc. Jpn. **80** (2011) 67002.

[35] Y. Ogasawara: *Mono no Mikata Toshiteno Isokukanron Nyumon* (Baifukan, Tokyo, 2011) [in Japanese].

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